Prethermalization of quantum systems interacting with non-equilibrium environments

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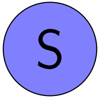
Model



Total Hamiltonian:

$$H = H_{\mathrm{S}}$$

Components:

$$H_{\mathrm{S}} = \frac{1}{2}\omega_0\sigma_z$$

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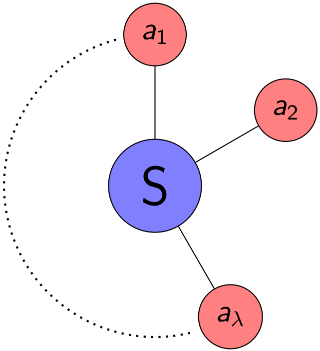

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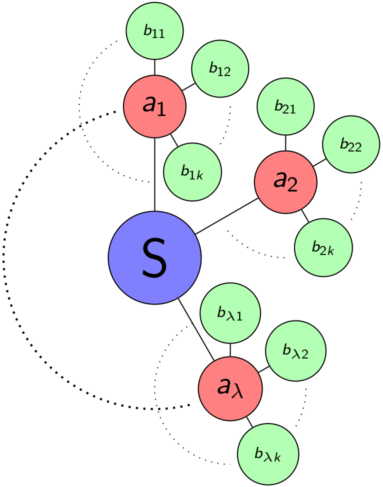

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$$H_{\mathrm{RII}} = \sum_{\lambda,k} \omega_{\lambda,k} b_{\lambda,k}^\dagger b_{\lambda,k} \quad H_{\mathrm{I2}} = \sum_{\lambda,k} g_{\lambda,k} a_\lambda b_{\lambda,k}^\dagger + h.c.$$

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- We model the spectral functions of the reservoirs with the phenomenological model:

$$J_i(\omega) = g_i \omega^{s_i} e^{-\omega/\omega_c} \ , \quad i = \mathrm{RI}, \mathrm{RII}$$

Methods

Redfield Master Equation

$$\frac{d}{dt}\rho_{\mathrm{S}}(t) = -i\left[H_{\mathrm{S}}, \rho_{\mathrm{S}}(t)\right] + \left(\int_0^t d\tau \alpha^{+}(t,\tau)\left[V_{\tau-t}\sigma_{+}\rho_{\mathrm{S}}(t), \sigma_{-}\right] \right.$$
$$\left. + \int_0^t d\tau \alpha^{-}(t,\tau)\left[V_{\tau-t}\sigma_{-}\rho_{\mathrm{S}}(t), \sigma_{+}\right] + h.c. \right) \,,$$

where $V_t\mathcal{O} = e^{iH_{\mathrm{S}}t}\mathcal{O}e^{-iH_{\mathrm{S}}t}$ represents the free evolution of the operator.

Methods

Canonical form of the Master Equation

$$\frac{d}{dt}\rho_{\mathrm{S}}(t) = -i[H(t), \rho_{\mathrm{S}}(t)] + \sum_{k=1}^{d^2-1} \gamma_k(t)\left(L_k(t)\rho_{\mathrm{S}}(t)L_k^\dagger(t) - \frac{1}{2}\{L_k^\dagger(t)L_k(t), \rho_{\mathrm{S}}(t)\}\right),$$

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with

$$\gamma_+(t) = J_{\mathrm{I}}(\omega_0)n_{\mathrm{I}}(\omega_0)e^{-J_{\mathrm{II}}(\omega_0)t} + J_{\mathrm{I}}(\omega_0)n_{\mathrm{II}}(\omega_0)(1-e^{-J_{\mathrm{II}}(\omega_0)t}) , \quad L_+ = \sigma_+$$

$$\gamma_-(t) = J_{\mathrm{I}}(\omega_0)(n_{\mathrm{I}}(\omega_0)+1)e^{-J_{\mathrm{II}}(\omega_0)t} + J_{\mathrm{I}}(\omega_0)(n_{\mathrm{II}}(\omega_0)+1)(1-e^{-J_{\mathrm{II}}(\omega_0)t}) , \quad L_- = \sigma_-$$

where

$$n_i(\omega) = \frac{1}{e^{-\beta_{\mathrm{i}}\omega}-1}$$

Results - Prethermalization

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1. Relaxation of any initial condition to a thermal state determined by the temperature of RI.

2. The system remains stationary in that state

3. Final relaxation towards a thermal state determined by the temperature of RII.

Results - Prethermalization

Animations!!

Result - Prethermalization time

Trace distance between the prethermal state and the evolved state of the system to study dependence of prethermalization time on other parameters.

$$T(\rho_S(t), \rho_S^{\mathrm{th}}(\beta_{\mathrm{I}})) = \frac{1}{2}\mathrm{Tr}\left\{\sqrt{(\rho_S(t) - \rho_S^{\mathrm{th}}(\beta_{\mathrm{I}}))^2}\right\} \tag{1}$$

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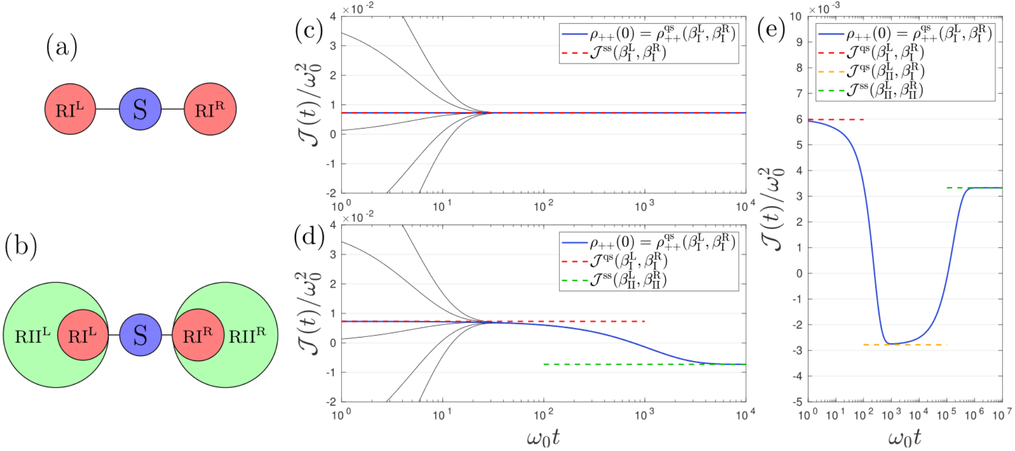
- Exponential dependence with the coupling strength between environments.
- When the temperatures between reservoirs are closer, it becomes larger.
- Hotter RI yields longer prethermalization times, as well as colder RII.

Results - Multiple environments



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- Model reminiscent of the layered structure of quantum computers, with different layers that are colder close to the qubits.